EQUATIONS OF MOTION

General Equations of Motion :—

$$v = \frac{dx}{dt} \Rightarrow dx = v\,dt \Rightarrow \int_{x_0}^{x(t)} dx = x(t) - x_0 = \int_{t_1}^{t_2} v\,dt$$

$$a = \frac{dv}{dt} \Rightarrow dv = a\,dt \Rightarrow \int_u^V dv = V - u = \int_{t_1}^{t_2} a\,dt$$

$$a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx} \Rightarrow v\,dv = a\,dx \Rightarrow \int_u^V v\,dv = \frac{1}{2}(V^2 - u^2) = \int_{x_1}^{x_2} a\,dx$$

Motion with Constant acceleration :—

Suppose a particle is at $x = x_0$ at time $t = 0$ & its speed be (velocity) u. Let the particle's acceleration be a which remains constant & its Velocity at time t be V.

$$\frac{dV}{dt} = a \Rightarrow dV = a\,dt \Rightarrow \int_u^V dV = \int_0^t a\,dt \Rightarrow V - u = at \Rightarrow V = u + at$$

$$\frac{dx}{dt} = V = u + at \Rightarrow \int_{x_0}^{x} dx = \int_0^t (u + at)\,dt \Rightarrow x - x_0 = ut + \tfrac{1}{2}at^2.$$

$$V^2 = (u + at)^2 = u^2 + 2uat + t^2a^2 = u^2 + 2a(ut + \tfrac{1}{2}at^2) = u^2 + 2ax$$

Distance travelled in n^{th} second — (Displacement) $S_{n^{th}} = S_{n+1} - S_n = \left[u + \tfrac{1}{2}a(2n - 1s)(1s)\right]$

- $x \rightarrow$ Position of the particle at time t i.e. displacement (not distance).

$$\begin{aligned} v &= u + at \\ s &= ut + \tfrac{1}{2}at^2 \\ v^2 &= u^2 + 2as \\ S_{n^{th}} &= u + \tfrac{a}{2}(2n - 1) \end{aligned}$$

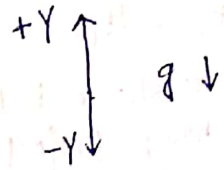
Restrictions !
(I) a = Const
(II) One dim. motion
(III) Inertial frame of ref.

FREELY FALLING BODIES :-

- Example of motion in a straight line with constant acceleration - Free fall of a body near earth's surface.

$a = -g$.

$v = u - gt$, $y = ut - \frac{1}{2}gt^2$, $v^2 = u^2 - 2gy$.

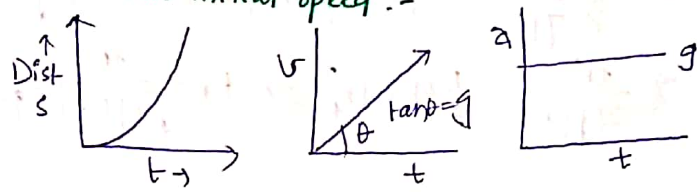


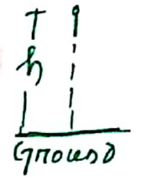
- **Body dropped from height 'h' with zero initial speed :-**

$t = \sqrt{\frac{2h}{g}}$, $V = \sqrt{2gh}$

$v(t) = gt$, $h(t) = gt^2/2$

$h_{n^{th}} = g/2\,(2n-1)$





- **Body projected vertically downward with initial vel. u :-**

$h(t) = ut + \frac{1}{2}gt^2$

$v^2(h) = u^2 + 2gh$, $v(t) = u + gt$

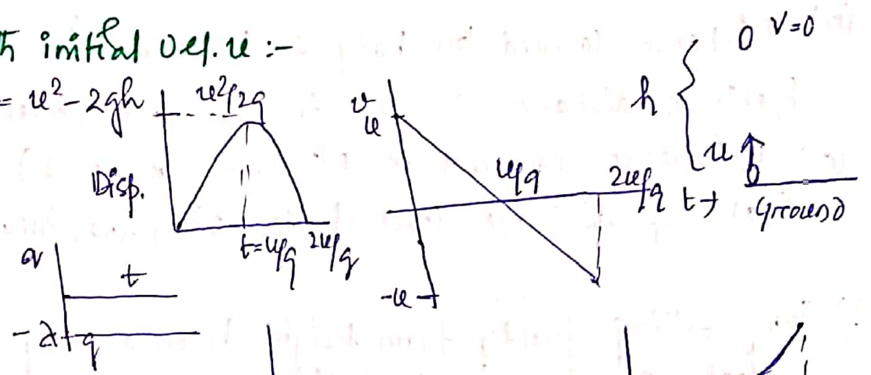
$h_{n^{th}} = u + g/2\,(2n-1)$

- **Body projected vertically upward with initial vel. u :-**

$v(t) = u - gt$, $h(t) = ut - \frac{1}{2}gt^2$; $v^2 = u^2 - 2gh$

$h_{n^{th}} = u - g/2\,(2n-1)$

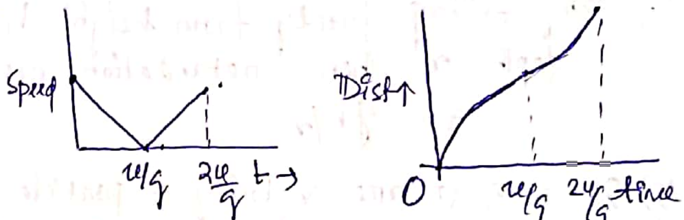
$t = \sqrt{\frac{2h}{g}} = \frac{u}{g}$, $u = \sqrt{2gh}$, $h = \frac{u^2}{2g}$.



- During complete journey -

Displacement $= 0$, $\vec{V}_{av} = 0$

Distance $= u^2/g$, $V_{av} = u/2$.



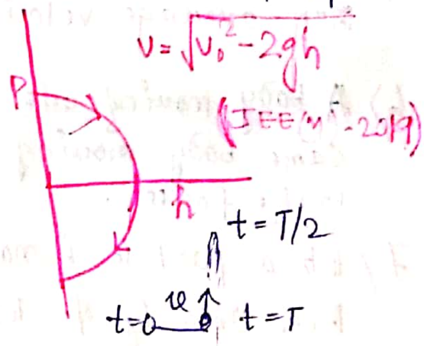
- If t = time it takes to reach highest point

Distance travelled in t^{th} sec = Dist travelled in $(t+1)^{th}$ sec.

$(t-1)^{th}$ = $(t+2)^{th}$

$(t-r)^{th}$ = $(t+r+1)^{th}$

- Time of ascent = Time of descent = Time of flight$/2$ = u/g.



- **Body projected upward from certain height :- (u) -**
- Speed when it acquires same level $= u$
- Speed at ground $v = \sqrt{u^2 + 2gh}$
- Time required to attain same level $= 2u/g$
- Total time of flight $T' = \left(T + \sqrt{T^2 + \frac{8h}{g}}\right)/2$.